

1. a) —

b) All you need to do is to make proper interpretations of the Nehari theorem and the similar theorem for Toeplitz operators.

2. a) —

b) One way to do it is to use Parseval's formula and then perform a conditional optimization (FlerDim/LTH level).

c) One possibility is to use f_{opt} from 2b) and the following inequality

$$\|Hf\|_2 = \|P_-(\phi - h)f\|_2 \leq \|(\phi - h)f\|_2 \leq \|\phi - h\|_\infty \|f\|_2.$$

For f_{opt} and h_{opt} we get

$$\|H\| = \|Hf_{opt}\|_2 = \|\phi - h_{opt}\|_\infty \|f_{opt}\|_2.$$

and, therefore, we have actually **equality** everywhere above (alignment principle in optimization). Use this fact to find h_{opt} .

3. a) —

b) Check out <http://bib.tiera.ru/b/90250>, page 165. A screenshot of it is available on the home page http://www.maths.lth.se/matematiklth/personal/ghulchak/regler_fan/2013/douglas.jpg