LUND UNIVERSITY
DEPARTMENT OF MATHEMATICS

FAN IN CONTROL
Hints 13
2013–12–02 kl 10–12

1. —

2. Non-trivial part $\Leftarrow$.
Approach 1: Set $F(x) = \sup_k(\langle f_k, x\rangle - c_k)$ and show that $K \neq \emptyset \Leftrightarrow \inf_{x\in X} F(x) > 0$. At a certain point you need to use the minimax theorem and to do so $\sup_k$ needs to be replaced with sup over another set that fits the minimax theorem assumptions.
Approach 2: Set $G = \{(y_1, \ldots . y_n) \in \mathbf{R}^n : \exists x \in X,\ \langle f_k, x\rangle - c_k \leq y_k, k = 1, \ldots, n\}$. Can you separate G and the origin by a hyperplane?

3. Non-trivial part $\Leftarrow$. Prove by contradiction that

$$x \in \bigcap_{\substack{x_0 \in \partial K \\ K \subset H_+}} H_+ \quad \Rightarrow x \in K.$$

4. —