LUND UNIVERSITY
DEPARTMENT OF MATHEMATICS

FAN IN CONTROL
Exercise 6
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1. Consider the linear system G on $(-\infty, +\infty)$

$$\begin{cases} \dot{x} &= Ax + Bu, \\ y &= Cx + Du \end{cases}$$

with the state vector $x \in \mathbf{R}^n$ and A having no eigenvalues on the imaginary axis.

a) Assuming A being anti-stable matrix, show that the 'future-to-past' Hankel operator $H\colon \mathbf{L}^{2+} \to \mathbf{L}^{2-}$ can be written as $\Gamma_o \Gamma_c$ where $\Gamma_c\colon \mathbf{L}^{2+} \to \mathbf{R}^n$ and $\Gamma_o\colon \mathbf{R}^n \to \mathbf{L}^{2-}$. Obtain the Lyapunov equations for $P_c = \Gamma_c \Gamma_c^*$ and $P_o = \Gamma_o^* \Gamma_o$.

b) Assuming A being stable matrix, repeate the Problem 1a) for the 'past-to-future' Hankel operator $\tilde{H}\colon \mathbf{L}^{2-} \to \mathbf{L}^{2+}$.

2. Consider the linear system G in the Problem 1 with the transfer function $\Phi_G(s)$.

a) If we change the time in G as $\tau = -t$, we get the system G_τ. Find the relation between Φ_G and Φ_{G_τ}.

b) Let G^* be the adjoint system to G. Show that $(G^*)_\tau = (G_\tau)^*$.

c) Define the transformation $G^\sim = G_\tau^*$. Find a state space realization for $G^\sim$ and the relation between the transfer functions $\Phi_{G^\sim}(s)$ and $\Phi_G(s)$.

d) Show that for the 'future-to-future' Toeplitz operators it holds $\|T_{\Phi_{G^\sim}}\| = \|T_{\Phi_G}\|$.

3. Finish the Problem 2b)+c) from the Exercise 5.