LUND UNIVERSITY
DEPARTMENT OF MATHEMATICS

FAN IN CONTROL
Exercise 5
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1. Consider a linear system[1] G with the transfer function $\phi \in L^\infty$. The system produces naturally two bounded operators: T ('future-to-future') and H ('future-to-past').

a) Show that adding any *stable* linear system to G does not affect H.

b) Show[2] that among all such systems in 1a) there is the best stable approximation of G in the sense that it minimizes the 'future-to-future' effect of the difference. What is the best approximation error?

2. Consider the linear system G: $y_k = 2u_{k+1} + u_{k+2}$, $k \in \mathbf{Z}$.

a) Write down the corresponding (infinite-dimentional) operator matrix for G. Extract the operator matrix for H and calculate its norm.

b) Calculate[3] $\|H\|$ by definition, i.e.

$$\|H\| = \sup_{\|u\|_2 \leq 1} \|Hu\|_2$$

and compare with 2a).

c) Find[4] the best stable approximation to G in the sense of 1b) (for example, provide the transfer function for it).

3. Consider a bounded Toeplitz operator $T_\phi = P_+ \phi P_+$.

a) Calculate T_ϕ^* and show that $\|T_\phi\| = \|T_\phi^*\|$.

b) Let $\phi \not\equiv 0$ (not identical zero). Show that either $\operatorname{Ker} T_\phi = 0$ or $\operatorname{Ker} T_\phi^* = 0$.[5]

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[1] It does not matter if it is a discrete-time or continuous-time system.
[2] The hint is available. However, try to solve it first by yourself.
[3] The hint is available.
[4] Difficult. The hint is available. If you cannot solve it after reading the hint — no trouble. Good to think on the problem anyway. We will discuss the solution.
[5] It is Coburn's lemma, 1966.