

# Computer Vision: Lecture 9

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# Today's Lecture

## Reconstruction and Optimization

- Objective Function: Reconstruction Error
- Principles of Local Optimization
- Least Squares Optimization
- Non-Linear Least Squares



# Minimizing Reprojection Error

Under the assumption that image points are corrupted by Gaussian noise, minimize the reprojection error.

## The reprojection error

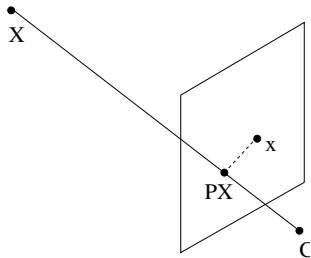
In regular coordinates  
( $\mathbf{x} = (x, y)$ ) the projection is

$$\left( \frac{P^1 \mathbf{X}}{P^3 \mathbf{X}}, \frac{P^2 \mathbf{X}}{P^3 \mathbf{X}} \right),$$

$P^1, P^2, P^3$  are the rows of  $P$ .

The reprojection error is

$$\left\| \left( x - \frac{P^1 \mathbf{X}}{P^3 \mathbf{X}}, y - \frac{P^2 \mathbf{X}}{P^3 \mathbf{X}} \right) \right\|^2.$$



# Minimizing Reprojection Error

## Calibrated Structure and Motion

Given image projections  $\{(x_{ij}, y_{ij})\}$  ( $i = \text{point nr}, j = \text{image nr}$ ), find 3D points  $\mathbf{X}_i$  and cameras  $P_j = [R_j \quad t_j]$  such that

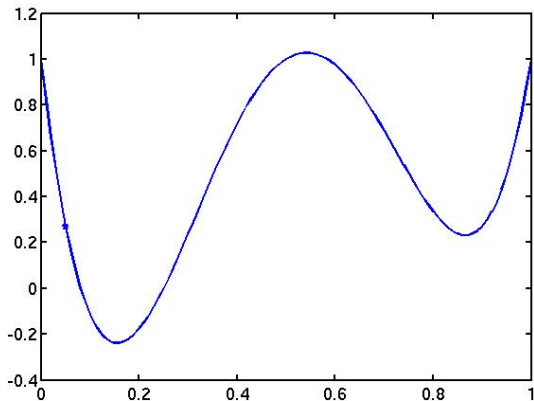
$$\sum_{ij} \left\| \left( x_{ij} - \frac{P_j^1 \mathbf{X}_i}{P_j^3 \mathbf{X}_i}, y_{ij} - \frac{P_j^2 \mathbf{X}_i}{P_j^3 \mathbf{X}_i} \right) \right\|^2,$$

is minimized.

- Complicated non linear expression.
- No closed form solution.



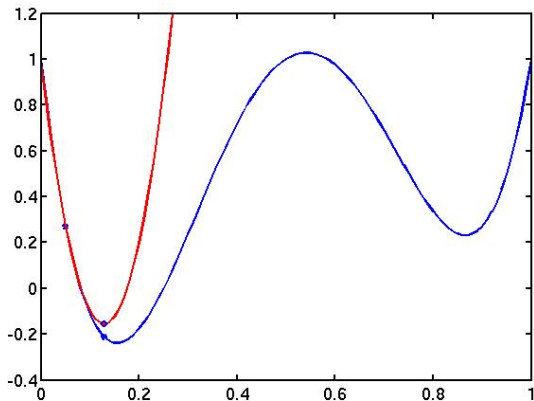
# Minimizing Reprojection Error, Locally



- Pick a starting point.



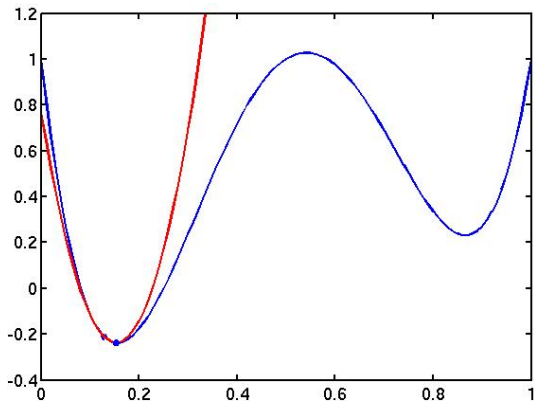
# Minimizing Reprojection Error, Locally



- Approximate the function using 2nd order Taylor expansion and minimize.



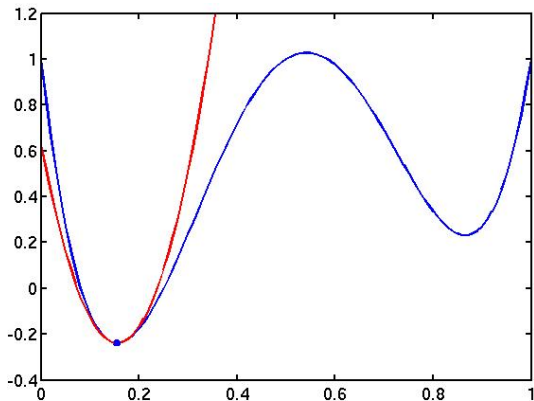
# Minimizing Reprojection Error, Locally



- Repeat.



# Minimizing Reprojection Error, Locally

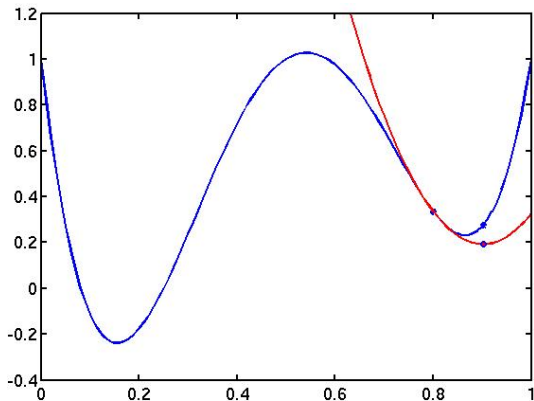


Newtons method.





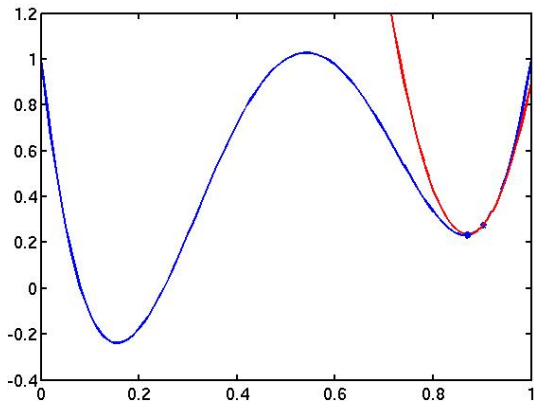
# Minimizing Reprojection Error, Locally



Different starting point.



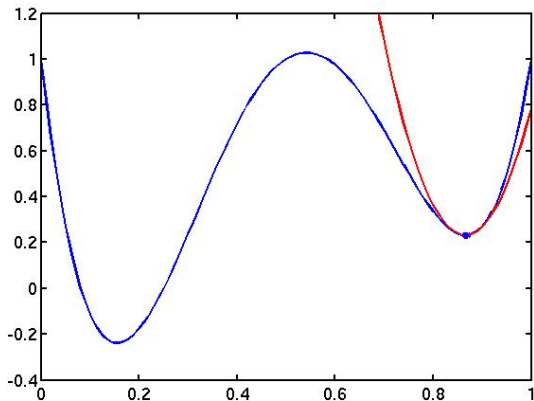
# Minimizing Reprojection Error, Locally



Different starting point.



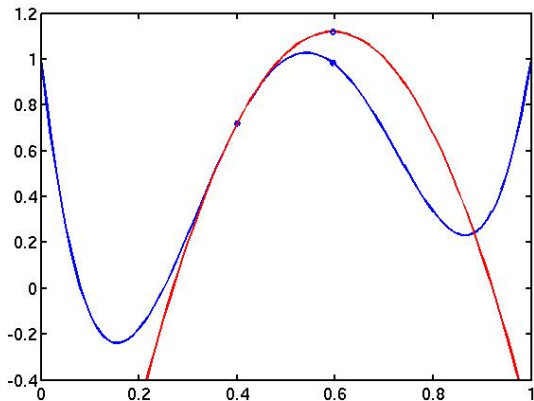
# Minimizing Reprojection Error, Locally



Leads to local minimum.



# Minimizing Reprojection Error, Locally



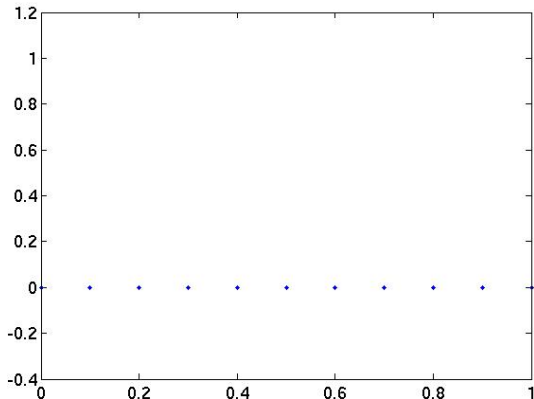
Third starting point, leads to local maximum.



# Minimizing Reprojection Error, Locally

Why not just sample the function?

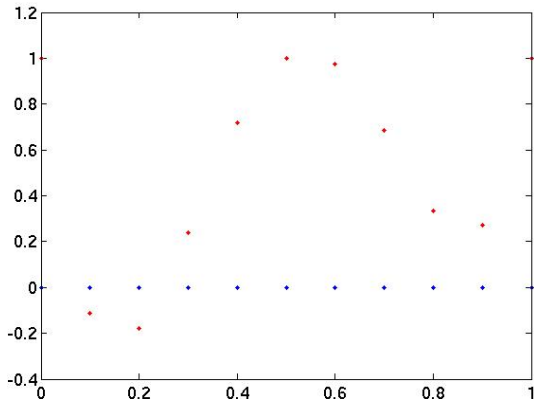
One dimensional function:



# Minimizing Reprojection Error, Locally

Why not just sample the function?

One dimensional function:



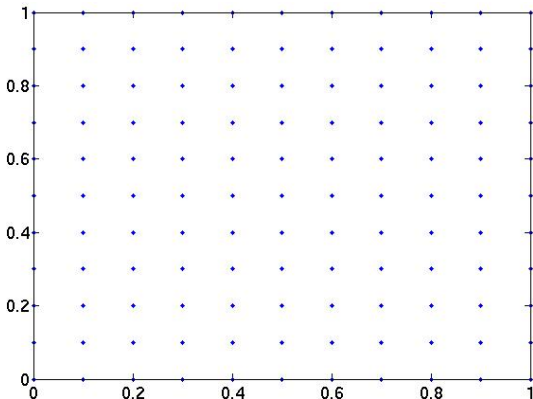
Sample 10 points, pick lowest value. Probably works.



# Minimizing Reprojection Error, Locally

Why not just sample the function?

Two dimensional function:



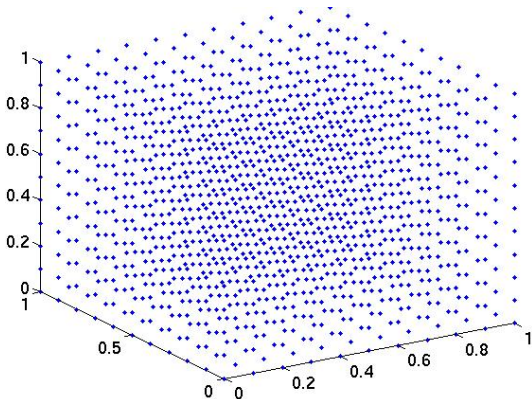
$10^2$  samples.



# Minimizing Reprojection Error, Locally

Why not just sample the function?

Three dimensional function:

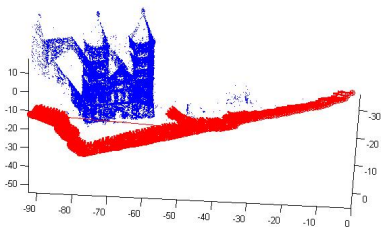


$10^3$  samples.





# Minimizing Reprojection Error, Locally



How many variables do we have?

The cathedral dataset:

- 480 camera matrices  $[R_i \ t_i]$ .  
Rotation part 3 dof, translation part 3 dof.  
Totally:  $480(3 + 3) = 2880$ .
- 91178 3D points.  
3 dof each.  
Totally:  $91178 \cdot 3 = 273534$



# Local Optimization

See lecture notes.



# Steepest Descent

Non-linear least squares:

$$\min_v \|r(v)\|^2$$

Linear approximation of residuals:

$$r(v) \approx r(v_0) + J(v_0)(v - v_0).$$

Line-search along the direction

$$d = -\frac{J(v_0)^T r(v_0)}{|J(v_0)^T r(v_0)|}$$

Demonstration...



# Gauss-Newton

Non-linear least squares:

$$\min_v \|r(v)\|^2$$

Linear approximation of residuals:

$$r(v) \approx r(v_0) + J(v_0)(v - v_0).$$

Line-search along the direction

$$\min \|r(v_0) + J(v_0)d\|^2.$$

Demonstration...



# Levenberg-Marquard

Non-linear least squares:

$$\min_v \|r(v)\|^2$$

Linear approximation of residuals:

$$r(v) \approx r(v_0) + J(v_0)(v - v_0).$$

Line-search along the direction

$$\min \|r(v_0) + J(v_0)d\|^2 + \lambda\|d\|^2.$$

Demonstration...

