## Today’s Lecture

### Repetition
- Reconstruction Problems
- A First Reconstruction System

### Model Fitting
- Line Fitting and Noise models
- Linear Least Squares
- Total Least Squares
- Outliers and Robust Estimation
- Reprojection error
Repetition: Computing the Camera Matrix

Known

- Image points $x_i$.

- Known scene points $X_i$.

Estimate

A camera matrix $P$ such that

$$\lambda_i x_i = P X_i.$$ 

Solved using DLT in lecture 3.
Repetition: Relative Orientation

Known:
Two corresponding point sets \( \{ \bar{x}_i \} \) and \( \{ x_i \} \).

Sought:
Scene points \( \{ X_i \} \) and cameras \( P_1 \) \( P_2 \), such that

\[
\lambda_i x_i = P_1 X_i \\
\bar{\lambda}_i \bar{x}_i = P_2 X_i
\]
The Fundamental Matrix (see lecture 5)

For cameras $P_1 = [I \ 0]$ and $P_2 = [A \ t]$. The corresponding image points $x_i$ and $\bar{x}_i$ fulfills

$$\bar{x}_i^T F x_i = 0,$$

where, $F = [t] \times A$.

- The scene point $X_i$ has been eliminated.
- Solve $F$ using 8-point alg, compute cameras (lect. 5).

Problem: Projective ambiguity
The Essential Matrix (see lecture 5)

For cameras $P_1 = [I \ 0]$ and $P_2 = [R \ t]$. The corresponding image points $x_i$ and $\bar{x}_i$ fulfills

$$\bar{x}_i^T E x_i = 0,$$

where, $F = [t] \times R$.

- The scene point $X_i$ has been eliminated.
- Solve $E$ using modified 8-point alg, compute cameras (lect. 6).

No projective ambiguity
Known

Image points \( \{x_{ij}\} \).

Camera matrices \( P_j \)

Sought

3D points \( X_i \), such that

\[
\lambda_{ij} x_{ij} = P_j X_i
\]

See lecture 4.
Sequential Reconstruction

Given lots of images

How do we compute the entire reconstruction?

1. For an initial pair of images, compute the cameras and visible scene points, using 8-point alg.
2. For a new image viewing some of the previously reconstructed scene points, find the camera matrix, using DLT.
3. Compute new scene points using triangulation.
4. If there are more cameras goto step 2.
A First Reconstruction System

Demonstrations…
A First Reconstruction System

Issues

- Outliers.
- Noise sensitivity.
- How to select initial pair.
- Unreliable 3D points.

Will get back to these issues later in the course.
Model Fitting

Given a set of model parameters find the parameter values that give the "best" fit the the data.

Examples:

**Camera Estimation** Given scene points $\mathbf{X}_i$ find $\mathbf{P}$ such that $\mathbf{PX}_i$ gives the best fit to the detected image points $\mathbf{x}_i$.

**Line Fitting** Find the line that best fits a set of 2D-points $(x_i, y_i)$.

What is the "best" fit? Depends on the noise model.
See lecture notes.
Least Squares Line Fitting

\[ \min \sum_i (ax_i + b - y_i)^2 \]

In matrix form

\[ \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \]

In matlab use

\[ A \backslash B \]
Total Least Squares Line Fitting

\[
\begin{align*}
\min & \quad \sum_i (ax_i + by_i + c)^2 \\
\text{s.t} & \quad a^2 + b^2 = 1
\end{align*}
\]

Let

\[
\bar{x} = \frac{1}{m} \sum_{i=1}^{m} x_i \quad \text{and} \quad \bar{y} = \frac{1}{m} \sum_{i=1}^{m} y_i
\]

The optimum fulfills

\[
\sum_{i=1}^{m} \begin{bmatrix}
(x_i - \bar{x})(x_i - \bar{x}) & (y_i - \bar{y})(x_i - \bar{x}) \\
(x_i - \bar{x})(y_i - \bar{y}) & (y_i - \bar{y})(y_i - \bar{y})
\end{bmatrix}
\begin{bmatrix}
 a \\
 b
\end{bmatrix} = \lambda
\begin{bmatrix}
 a \\
 b
\end{bmatrix}
\]

and

\[
c = -(a\bar{x} + b\bar{y})
\]
Outliers and Robust Loss Functions
We want to remove measurements that do not obey the Gaussian-noise model before doing least squares fitting.
Outliers and Robust Loss Functions

See lecture notes.
Outliers and Robust Loss Functions

\[ \rho_1(\epsilon_i^2) = \epsilon_i^2 \]

\[ \rho_2(\epsilon_i^2) = \min(\epsilon_i^2, \tau^2) \]

\[ \rho_3(\epsilon_i^2) = \frac{\ln(1+\tau^2) - \ln(e^{-\epsilon_i^2} + \tau^2)}{\ln(1+\tau^2)} \]

\[ \rho_4(\epsilon_i^2) = b^2 \ln(1 + \frac{\epsilon_i^2}{b^2}) \]

\[ \rho_5(\epsilon_i^2) = \begin{cases} 2b|\epsilon_i| - b^2 & |\epsilon_i| \geq b \\ \epsilon_i^2 & |\epsilon_i| \leq b \end{cases} \]

\[ \rho_1'(\epsilon_i^2) = 1 \]

\[ \rho_2'(\epsilon_i^2) = \begin{cases} 0 & |\epsilon_i| > \tau \\ 1 & |\epsilon_i| \leq \tau \end{cases} \]

\[ \rho_3'(\epsilon_i^2) = \frac{e^{-\epsilon_i^2}}{(e^{-\epsilon_i^2} + \tau^2)\ln(1+\tau^2)} \]

\[ \rho_4'(\epsilon_i^2) = \frac{b^2}{b^2 + \epsilon_i^2} \]

\[ \rho_5'(\epsilon_i^2) = \begin{cases} \frac{b}{|\epsilon_i|} & |\epsilon_i| \geq b \\ 1 & |\epsilon_i| \leq b \end{cases} \]
Gaussian Noise

When outliers have been removed, measurements are still corrupted by noise. The exact position of a feature may be difficult to determine.
Under the assumption that image points are corrupted by Gaussian noise, minimize the reprojection error.

**The reprojection error**

In regular coordinates the projection is

\[
\begin{pmatrix}
\frac{p^1_x}{p^3_x} & \frac{p^2_x}{p^3_x}
\end{pmatrix},
\]

\[P^1, P^2, P^3\] are the rows of \(P\).

The reprojection error is

\[
\| \begin{pmatrix}
x_1 - \frac{p^1_x}{p^3_x} \\
x_2 - \frac{p^2_x}{p^3_x}
\end{pmatrix} \|^2.
\]
Demonstration ...
Reprojection Error vs. Algebraic Error

Algebraic Error
Attempts to find an approximate solution to an algebraic equation. Ex. DLT
\[ \min \sum_i \| \lambda_i x_i - PX_i \|^2, \]
8-point algorithm etc.

Reprojection Error
- Gives most probable solution (least squares).
- Geometrically meaningful.
- Nonlinear equations, difficult to optimize. Often requires starting solutions.

Algebraic Error
- No clear geometrical meaning.
- May produce poor solutions.
- Easy to optimize, using e.g. svd.

Use algebraic solution as starting solution (next lecture).
To do

- Work on assignment 4