Computer Vision: Lecture 6

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Today's Lecture

Two view geometry

- Computing cameras from F
- The calibrated case: The Essential Matrix
- The 8-point algorithm (again)
- Computing the cameras from E.

Computing Cameras from F

See Lecture notes....

Exercise 1

What is the camera center of $P_2 = [[e_2]_\times F \quad e_2]$? (Hint: Recall that $Fe_1 = 0$.)

Exercise 2

Verify that the projections in $P_1 = [I \quad 0]$ and $P_2 = \left[[e_2]_\times F + e_2 v^T \quad \lambda e_2\right]$ fulfill the epipolar constraints for any $v \in \mathbb{R}^3$ and $\lambda \neq 0$.

Computing Cameras from F

Demo.

Relative Orientation: The Calibrated Case

Problem Formulation

Given two sets of corresponding (normalized) points $\{\mathbf{x}_i\}$ and $\{\bar{\mathbf{x}}_i\}$, compute camera matrices $P_1 = \begin{bmatrix} R_1 & t_1 \end{bmatrix}$, $P_2 = \begin{bmatrix} R_2 & t_2 \end{bmatrix}$ and 3D-points $\{\mathbf{X}_i\}$ such that

$$\lambda_i \mathbf{x}_i = P_1 \mathbf{X}_i$$

and

$$\bar{\lambda}_i \bar{\mathbf{x}}_i = P_2 \mathbf{X}_i.$$

Relative Orientation: Problem Formulation

Simplification

If $P_1 = \begin{bmatrix} R_1 & t_1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} R_2 & t_2 \end{bmatrix}$, apply the transformation

$$H = \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0 & 1 \end{bmatrix}.$$

Then

$$P_1 H = \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}.$$

Hence, we may assume that the cameras are

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} R & t \end{bmatrix}$$

The Essential Matrix

The Essential Matrix

The camera pair $P_1 = \begin{bmatrix} I & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} R & t \end{bmatrix}$ has the fundamental matrix

$$E = [t]_\times R.$$

E is called the essential matrix.

- R has 3 dof, t 3 dof, but the scale is arbitrary, therefore E has 5 dof.
- E has $det(E) = 0$
- E has two nonzero equal singular values.

The Essential Matrix

The 8-point algorithm (again)

- Extract at least 8 point correspondences.
- Normalize the coordinates (multiply with K^{-1}, K inner parameters).
- Form M and solve
$$\min_{||v||^2=1} ||Mv||^2,$$
using svd.
- Form the matrix E (ensure that $det(E) = 0$ and that E has two nonzero equal singular values).
- Compute a pair of cameras from E.
- Compute the scene points.

The Essential Matrix

Issues

Resulting E may not have $det(E) = 0$ and two nonzero equal singular values.
Pick the closest essential matrix A.

Can be solved using svd, in matlab:

$$
\begin{aligned}
&[U, S, V] = svd(E);\\
&s = (S(1,1) + S(2,2))/2;\\
&S = diag([s\ s\ 0]);\\
&A = U * S * V';
\end{aligned}
$$

Note: Since the scale of the essential matrix is arbitrary we may assume that $s = 1$. That is use $S = diag([1\ 1\ 0]);$ instead.

Computing the cameras

Want to find $P_2 = [R\ t]$ such that $E = [t]_\times R$.
Outline:

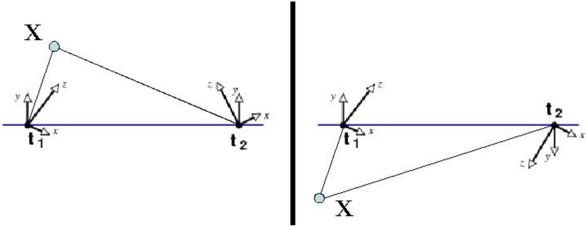
- Ensure that E has the SVD

$$E = USV^T$$

  where $det(UV^T) = 1$.
- Compute a factorization $E = SR$ where S is skew symmetric and R a rotation.
- Compute a t such that $[t]_\times = S$.
- Form the camera $P_2 = [R\ t]$.

See lecture notes for details...

The Twisted Pair



4 Solutions


XEROX
BUSINESS
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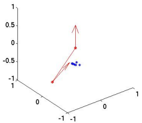
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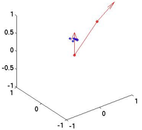
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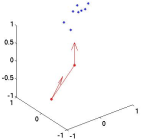
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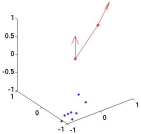
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To do

- Work on assignment 3