Two view geometry

- Computing cameras from $F$
- The calibrated case: The Essential Matrix
- The 8-point algorithm (again)
- Computing the cameras from $E$. 
See Lecture notes....
Computing Cameras from F

Demo.
Problem Formulation

Given two sets of corresponding (normalized) points \( \{x_i\} \) and \( \{\bar{x}_i\} \), compute camera matrices \( P_1 = \begin{bmatrix} R_1 & t_1 \end{bmatrix} \), \( P_2 = \begin{bmatrix} R_2 & t_2 \end{bmatrix} \) and 3D-points \( \{X_i\} \) such that

\[ \lambda_i x_i = P_1 X_i \]

and

\[ \bar{\lambda}_i \bar{x}_i = P_2 X_i. \]
If $P_1 = \begin{bmatrix} R_1 & t_1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} R_2 & t_2 \end{bmatrix}$, apply the transformation

$$H = \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0 & 1 \end{bmatrix}.$$ 

Then

$$P_1 H = \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} R_1^T & -R_1^T t_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}.$$ 

Hence, we may assume that the cameras are

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} R & t \end{bmatrix}.$$
The Essential Matrix

The camera pair $P_1 = [I \ 0]$ and $P_2 = [R \ t]$ has the fundamental matrix

$$E = [t] \times R.$$ 

$E$ is called the essential matrix.

- $R$ has 3 dof, $t$ 3 dof, but the scale is arbitrary, therefore $E$ has 5 dof.
- $E$ has $det(E) = 0$
- $E$ has two nonzero equal singular values.
The Essential Matrix

The 8-point algorithm (again)

- Extract at least 8 point correspondences.
- Normalize the coordinates (multiply with $K^{-1}$, $K$ inner parameters).
- Form $M$ and solve

$$
\min_{\|v\|^2=1} \|Mv\|^2,
$$

using svd.
- Form the matrix $E$ (ensure that $\det(E) = 0$ and that $E$ has two nonzero equal singular values).
- Compute a pair of cameras from $E$.
- Compute the scene points.
Issues

Resulting $E$ may not have $\text{det}(E) = 0$ and two nonzero equal singular values.

Pick the closest essential matrix $A$.

Can be solved using svd, in matlab:

$$[U, S, V] = \text{svd}(E);$$
$$s = (S(1, 1) + S(2, 2))/2;$$
$$S = \text{diag}([s \ s \ 0]);$$
$$A = U \ast S \ast V';$$

Note: Since the scale of the essential matrix is arbitrary we may assume that $s = 1$. That is use $S = \text{diag}([1 \ 1 \ 0]);$ instead.
Computing the cameras

Want to find $P_2 = [R \, t]$ such that $E = [t]_\times R$.

Outline:
- Ensure that $E$ has the SVD
  \[ E = USV^T \]
  where $\det(UV^T) = 1$.
- Compute a factorization $E = SR$ where $S$ is skew symmetric and $R$ a rotation.
- Compute a $t$ such that $[t]_\times = S$.
- Form the camera $P_2 = [R \, t]$.

See lecture notes for details...
The Twisted Pair
To do

- Work on assignment 3