Computer Vision: Lecture 5

Carl Olsson

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Two view geometry

- Relative orientation of two cameras
- The epipolar constraints
- The uncalibrated case: The Fundamental Matrix
- The 8-point algorithm
Relative Orientation: Problem Formulation

Given

Two images and corresponding points.

Compute

The structure (3D-points) and the motion (camera matrices).
Mathematical Formulation

Given two sets of corresponding points \( \{x_i\} \) and \( \{\bar{x}_i\} \), compute camera matrices \( P_1 \), \( P_2 \) and 3D-points \( \{X_i\} \) such that

\[
\lambda_i x_i = P_1 X_i
\]

and

\[
\bar{\lambda}_i \bar{x}_i = P_2 X_i.
\]
Ambiguities (uncalibrated case)

Can always apply a projective transformation $H$ to archive a different solution

$$\lambda_i \mathbf{x}_i = P_1 HH^{-1} \mathbf{x}_i = P_1 \tilde{\mathbf{x}}_i$$

and

$$\bar{\lambda}_i \bar{\mathbf{x}}_i = P_2 HH^{-1} \mathbf{x}_i = P_2 \tilde{\mathbf{x}}_i.$$
If \( P_1 = [A_1 \ t_1] \) and \( P_2 = [A_2 \ t_2] \), apply the transformation

\[
H = \begin{bmatrix}
A_1^{-1} & -A_1^{-1}t_1 \\
0 & 1
\end{bmatrix}.
\]

Then

\[
P_1H = [A_1 \ t_1] \begin{bmatrix}
A_1^{-1} & -A_1^{-1}t_1 \\
0 & 1
\end{bmatrix} = [I \ 0].
\]

Hence, we may assume that the cameras are

\[
P_1 = [I \ 0] \quad \text{and} \quad P_2 = [A \ t]
\]
Consider a single point $x$ in the first image. Any point on the line projects to this point.
Any point on the projection of the 3D line can correspond to $x$. 
The projected lines should all meet in a point. The so called epipole is the projection of the camera center of the other camera.
The epipole $e_1$ is the projection of the $C_2$ in $P_1$. The epipole $e_2$ is the projection of the $C_1$ in $P_2$. $e_1, e_2$ usually outside field of view.
See lecture notes.
The Fundamental Matrix

Estimating $F$

If $x_i$ and $\bar{x}_i$ corresponding points

$$\bar{x}_i^T F x_i = 0.$$

If $x_i = (x_i, y_i, z_i)$ and $\bar{x}_i = (\bar{x}_i, \bar{y}_i, \bar{z}_i)$ then

$$\bar{x}_i^T F x_i = F_{11} \bar{x}_i x_i + F_{12} \bar{x}_i y_i + F_{13} \bar{x}_i z_i$$
$$+ F_{21} \bar{y}_i x_i + F_{22} \bar{y}_i y_i + F_{23} \bar{y}_i z_i$$
$$+ F_{31} \bar{z}_i x_i + F_{32} \bar{z}_i y_i + F_{33} \bar{z}_i z_i$$
The Fundamental Matrix

Estimating $F$

In matrix form (one row for each correspondence):

$$
\begin{bmatrix}
\bar{x}_1 x_1 & \bar{x}_1 y_1 & \bar{x}_1 z_1 & \ldots & \bar{z}_1 z_1 \\
\bar{x}_2 x_2 & \bar{x}_2 y_2 & \bar{x}_2 z_2 & \ldots & \bar{z}_2 z_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\bar{x}_n x_n & \bar{x}_n y_n & \bar{x}_n z_n & \ldots & \bar{z}_n z_n
\end{bmatrix}
\begin{bmatrix}
F_{11} \\
F_{12} \\
F_{13} \\
\vdots \\
F_{33}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
$$

Solve using homogeneous least squares (svd).

$F$ has 9 entries (but the scale is arbitrary). Need at least 8 equations (point correspondences).
Resulting $F$ may not have $\det(F) = 0$. 
Pick the closest matrix $A$ with $\det(A) = 0$.

Can be solved using svd, in matlab:

$$[U, S, V] = \text{svd}(F);$$
$$S(3, 3) = 0;$$
$$A = U \ast S \ast V';$$
The Fundamental Matrix

Issues

Normalization needed (see DLT).

If $x_1$ and $\bar{x}_1 \approx 1000$ pixels, the coefficients $z_1 \bar{z}_1 = 1$, $x_1 \bar{z}_1 = 1000$ and $x_1 \bar{x}_1 = 1000000$. May give poor numerics.

Not normalized:  

Normalized:
The Fundamental Matrix

The 8-point algorithm

- Extract at least 8 point correspondences.
- Normalize the coordinates (see DLT).
- Form $M$ and solve
  \[
  \min_{\|v\|^2=1} \|Mv\|^2,
  \]
  using svd.
- Form the matrix $F$ (ensure that $\det(F) = 0$).
- Transform back to the original coordinates.
- Compute a pair of cameras from $F$ (next lecture).
- Compute the scene points (next lecture).
The Fundamental Matrix

Demo.
To do

- Start working on Assignment 3.
- Search for "The Fundamental Matrix Song" on youtube.