Today's Lecture

Projective Geometry

- Homogeneous coordinates, vanishing points.
- Lines in $\mathbb{P}^2$/Planes in $\mathbb{P}^3$.
- Conics
- Projective transformations.
Homogeneous coordinates

See lecture notes...
Vanishing points and lines

The last supper by Duccio (around 1310).
The last supper by Duccio (around 1310). Parallel lines do not meet at a single vanishing point.
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Vanishing points

The last supper by da Vinci (1499).
Vanishing points

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Vanishing points

The last supper by da Vinci (1499).
Conics

See lecture notes...
Example: Point Transfer via a Plane.

If a set of points $U_i$ lying on the same plane is projected into two cameras $x_i \sim P_1 U_i$, $y_i \sim P_2 U_i$, then there is a homography such that $x_i \sim Hy_i$. 
Example: Point Transfer via a Plane.

Compute the homography by selecting (at least) 4 points, and solving $\lambda_i x_i = H y_i$. 
Projective Transformations

Example: Point Transfer via a Plane.

Apply transformation to right image.
Example: Point Transfer via a Plane.

Mean value of the two images. Points on the plane seem to agree.
Affine Transformations ($\mathbb{P}^n \rightarrow \mathbb{P}^n$)

$$H = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix},$$

where $A - n \times n$ (invertible) and $t - n \times 1$.

- Parallel lines are mapped to parallel lines.
- Preserves the line at infinity (points at infinity are mapped to points at infinity, and regular points are mapped to regular points).
- Can be written $y = Ax + t$ for points in $\mathbb{R}^n$. 

Carl Olsson
Computer Vision: Lecture 2
**Similarity Transformations** ($\mathbb{P}^n \rightarrow \mathbb{P}^n$)

\[
H = \begin{bmatrix} sR & t \\ 0 & 1 \end{bmatrix},
\]

where $R$ is an $n \times n$ rotation, $t$ is a $n \times 1$, and $s$ is a positive number.

- Special case of affine transformation.
- Preserves angles between lines.
Euclidian Transformations (Rigid body motion $\mathbb{P}^n \to \mathbb{P}^n$)

$$H = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix},$$

where $R - n \times n$ rotation, $t - n \times 1$.

- Special case of similarity.
- Preserves distances.
To do

- Work on assignment 1