The solutions should be handed in no later than 48 hours after collecting the exam. You may use any books and computer programs (e.g., Matlab and Maple). It is not permitted to get help from other persons. Credits can be given for partially solved problems. Write your solutions neatly and explain your calculations. Programs and long calculations can be submitted directly to calle@maths.lth.se. Both the content and the format of your solutions, and the difficulty of the problems solved, will affect your grade. For grade 4 or grade 5, it is not necessary to have solved four or five problems correctly, respectively, but it is sufficient.

1. Compute the projection of the scene points \((1, 0, 0)\), \((1, 1, 0)\) and \((0, 0, 1)\) in the camera

\[
P = \begin{bmatrix} 2 & 1 & 0 & -10 \\ 0 & 2 & 0 & -20 \\ 0 & 0 & 2 & 2 \end{bmatrix}.
\]

What happens if you try to compute the projection of \((0, 10, -1)\), and why?

2. Consider the projective transformations \(\mathbb{P}^2 \to \mathbb{P}^2\)

\[
H_1 = \begin{bmatrix} 2 & -1 & 8 \\ 1 & 2 & 10 \\ 0 & 0 & 4 \end{bmatrix} \quad \text{and} \quad H_2 = \begin{bmatrix} 2 & -1 & 8 \\ 1 & 2 & 10 \\ 1 & 0 & 4 \end{bmatrix}.
\]

(a) What type of projective transformations are these?

(Euclidean, similarity, affine or projective. Motivate your answer.)

(b) Transform the points \((-4, 0)\), \((0, 1)\) and \((2, 2)\) using \(H_1\) and \(H_2\). Interpret the results as regular points in \(\mathbb{R}^2\).

(c) Show that if

\[H_1 \mathbf{x} \sim H_2 \mathbf{x},\]

then

\[H \mathbf{x} = \lambda \mathbf{x},\]

where \(H = H_2^{-1} H_1\). That is \(\mathbf{x}\) is an eigenvector of \(H\).

(d) Find all points for which the transformation using \(H_1\) is the same as that of \(H_2\).
3. The image below shows the Rubik’s cube. The image is also available from www.maths.lth.se/matematiklth/vision/datorseende/datorseendell/rubik.jpg

a) Using a model of the cube (it is a cube with sides 5.5 cm) estimate the camera matrix $P$. Explain your model, particularly the choice of coordinate system in the scene. (Hint: The matlab command `ginput` may be useful.)

b) Compute the internal and external parameters. Are the results reasonable?

4. In the 8-point algorithm the problem of estimating the camera motion and the scene structure is solved using 8 points. In this exercise we will see that 7 points is actually enough.

a) Show that using 7 point correspondences in two images one gets 7 linear constraints for the elements of the fundamental matrix $F$. Show that the set of all matrices that fulfills these 7 equations can be written

$$F_0 + \lambda F_1,$$

where $F_0$ and $F_1$ are $3 \times 3$ matrices and $\lambda$ is a number.

b) Use that any fundamental matrix $F$ satisfies the constraint $\det(F) = 0$, to derive an equation for $\lambda$. Show that this equation is a third degree polynomial in $\lambda$.

c) Ignoring scale factors, how many possible fundamental matrices are there that work for a set of 7 point correspondences?

d) In the file www.maths.lth.se/vision/datorseende/datorseendell/Fpoints.m is two matrices x1 and x2 containing 7 point correspondences. Use these to form a matrix $M$ such that $Mf = 0$, where $f$ is the 9 entries of the fundamental matrix. Compute $F_0$ and $F_1$ using svd and find the $\lambda$’s that give determinant 0. Verify that the point correspondances fulfill the epipolar equations for all three solutions. (Hints: To solve $\det(F_0 + \lambda F_1) = 0$, without computing the polynomial by hand there are several options. If $F_1$ is invertible one can solve the eigenvalue problem $-F_1^{-1}F_0x = \lambda x$. Another option is to evaluate $\det(F_0 + \lambda F_1)$ for four different $\lambda$’s and and fit a polynomial $a\lambda^3 + b\lambda^2 + c\lambda + d$ and then use the `roots` command.)
5. The file

www.maths.lth.se/vision/datorseende/datorseende11/door.jpg
contains the following image:

![Image](www.maths.lth.se/vision/datorseende/datorseende11/door.jpg)

The black sign, to the right on the door, is rectangular with height 28 cm and width 25.5 cm. What is the length of the diagonal of the poster? (Hint: The matlab command `ginput` might be useful.)

6. A 3D point $X$ is being projected into a camera $P$. The vector between the projection and the measured 2D point $(x, y)$ is

$$v(X) = (v_1(X), v_2(X)) = \left( x - \frac{P^1 X}{P^3 X}, y - \frac{P^2 X}{P^3 X} \right),$$

where $P^1, P^2, P^3$ are the rows of $P$.

a) Show that a halfspace

$$\{X; a^T X \leq b\}$$

is a convex set.

b) Show that the set

$$\{X; |v_1(X)| \leq \epsilon, |v_2(X)| \leq \epsilon, P^3 X > 0\}$$

is a convex set.

c) The file

www.maths.lth.se/vision/datorseende/datorseende11/triang.mat

Contains 3 cameras, and three projections of a 3D point. Check if there is a 3D-point $X$ for which $|v_1(X)|$ and $|v_2(X)|$ is less than 2.5 pixels in all cameras. (Hint: The matlab command `linprog` may be helpful. To achieve numerical stability you might have to add the extra constraint $X_4 = 1$, where $X_4$ is the fourth coordinate of $X$.)

*Good Luck!*