The task is to write a short (around three pages) report on the topic below. The purpose of your report is to get a biology friend interested in it who is slightly averse to mathematics, to demonstrate that mathematical models can make some interesting predictions. The report should be written in the context of the dynamics of some animal population.

Below you will find a list of exercises to carry out in order to get acquainted with the material. They are mainly computer experiments and can be carried out in almost any programming language.

The project will first discuss the behaviour of the long-term solution to the recursion equation

\[ u_{n+1} = u_n e^{r(1-u_n)} \]

for different values of \( r > 0 \).

1. Investigate the long-term solution using simulations for different \( r \)'s

2. Make a plot which shows what values are obtained for large values of \( n \) (ignoring the transients – you may choose \( n = 1000 \) as the cut-off). The plot should have \( r \) on the \( x \)-axis and the long-term value on the \( y \)-axis.

3. Compute for each value of \( r \) an estimate of the entropy, defined as follows. Divide the data into \( M \) subintervals and let \( f_i \) be the observed frequency of observations in the \( i^{th} \) interval. The entropy is defined as
the number
\[ H(f_1, \ldots, f_M) = -\frac{1}{\ln M} \sum_{j=1}^{M} f_j \ln f_j \]
(term defined to be zero when \( f_j = 0 \)). Explain what this number measures and plot. How does it relate to the previous graph?

Based on your results, write up a report on this.

Then add a section to the report on the corresponding analysis of the following system, which is a non-dimensional version of a system describing the regulation of a host population by a microparasite population \( u_n \):

\[ u_{n+1} = ru_n (1 - I_n) \text{ where } 1 - I_n = e^{-I_n u_n}. \]

Here \( I_n \) denotes the fraction of the host population which has been infected by the time the epidemic has run its course. The assumption is that the parasite epidemic has spread through each generation before the next population count.

Please remember that the important aspect of the report is that it is readable and the discussion is in a biological context.