Formelblad för Funktionsteori och Komplex analys

Fourierserier

\[ f(t) \approx \sum_{k=-\infty}^{\infty} c_k e^{ik\Omega t} = c_0 + \sum_{k=1}^{\infty} a_k \cos k\Omega t + b_k \sin k\Omega t \quad \Omega T = 2\pi \]

\[ c_k = \frac{1}{T} \int_{\text{period}} e^{-ik\Omega t} f(t) \, dt \]

\[ \begin{cases} a_k = \frac{2}{T} \int_{\text{period}} \cos(k\Omega t) f(t) \, dt \\ b_k = \frac{2}{T} \int_{\text{period}} \sin(k\Omega t) f(t) \, dt \end{cases} \]

\[ \begin{cases} a_k = c_k + c_{-k} \\ b_k = i(c_k - c_{-k}) \end{cases} \]

\[ \begin{cases} c_k = \frac{1}{2}(a_k - ib_k) \\ c_{-k} = \frac{1}{2}(a_k + ib_k) \end{cases} \]

Parsevals formul

\[ \frac{1}{T} \int_{\text{period}} f(t)g(t) \, dt = \sum_{k=-\infty}^{\infty} c_k(f)c_k(g) \]

\[ \frac{1}{T} \int_{\text{period}} |f(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |c_k(f)|^2 \]

\[ \frac{1}{T} \int_{\text{period}} |f(t)|^2 \, dt = |c_0|^2 + \frac{1}{2} \sum_{k=1}^{\infty} (|a_k|^2 + |b_k|^2) \]

Halvperiodutvecklingar

Cosinusserie

\[ f \sim c_0 + \sum_{k=1}^{\infty} a_k \cos\left(\frac{k\pi}{L}x\right) \]

\[ a_k = \frac{2}{L} \int_{0}^{L} f(x) \cos\left(\frac{k\pi}{L}x\right) \, dx \]

\[ c_0 = \frac{1}{L} \int_{0}^{L} f(x) \, dx \]

Sinusserie

\[ f \sim \sum_{k=1}^{\infty} \beta_k \sin\left(\frac{k\pi}{L}x\right) \]

\[ \beta_k = \frac{2}{L} \int_{0}^{L} f(x) \sin\left(\frac{k\pi}{L}x\right) \, dx \]

Kvot- och rotkriteriet

\[ \kappa = \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| \]

\[ \kappa < 1 \implies \sum_{k} a_k \text{ konvergent} \quad \kappa > 1 \implies \sum_{k} a_k \text{ divergent} \]

\[ \rho = \lim_{k \to \infty} \sqrt[k]{|a_k|} \]

\[ \rho < 1 \implies \sum_{k} a_k \text{ konvergent} \quad \rho > 1 \implies \sum_{k} a_k \text{ divergent} \]

Potensserier

\[ (1 + x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k \]

\[ (1 - x)^{-n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} x^k \]

\[ e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k \]

\[ \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k} \]

\[ \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} \]

\[ \ln(1 + x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k \]

\[ \ln(1 + x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k} x^k \]

\[ \arctan x = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1} \]

\[ f(z) = \sum_{k=0}^{\infty} c_k (z - a)^k \]

\[ c_k = \frac{f^{(k)}(a)}{k!} = \frac{1}{2\pi i} \int_{C} \frac{f(w)}{(w - a)^{k+1}} \, dw \]
Residyregler

1. Om \( f(z) = (z - a)^{-N}g(z) \) så är \( \text{Res}_{z=a} f(z) = \frac{g^{(N-1)}(a)}{(N-1)!} \)

2. Om \( f(z) = (z - a)^{-N}g(z) \) och \( g(z) = \sum_{k=0}^{\infty} c_k (z - a)^k \) så är \( \text{Res}_{z=a} f(z) = c_{N-1} \)

3. \( \text{Res}_{z=a} f(z) = \lim_{z \to a} (z - a)f(z) \)

4. \( \frac{\text{Res}_{z=a} f_1(z)}{f_2(z)} = \frac{f_1(a)}{f_2(a)} \)

Funktionsserier

\[
\begin{align*}
|u_k(t)| & \leq m_k, \ t \in I \\
\sum_k m_k \text{ konvergent} \} & \implies \sum_k u_k(t) \text{ likformigt konvergent på } I \\
\sum_k u_k(t) \text{ likformigt konvergent} & \implies \sum_k u_k(t) \text{ kontinuerlig} \\
\sum_k u_k(t) \text{ konvergent} & \implies \sum_k u'_k(t) \text{ likformigt konvergent} \\
\sum_k u_k(t) \text{ likformigt konvergent på } I & \implies \int_I \left( \sum_k u_k(t) \right) dt = \sum_k \int_I u_k(t) dt \\
\text{u_k kontinuerliga, I begränsad} \} & \implies d \left( \sum_k u_k(t) \right) = \sum_k u'_k(t)
\end{align*}
\]

Räkneregler för potensserier

\[
\begin{align*}
A(x) + B(x) & \leftrightarrow a + b \\
xA(x) & \leftrightarrow \langle a_{k-1} \rangle \\
x \frac{dA}{dx} & \leftrightarrow \langle (k+1)a_{k+1} \rangle \\
(1 - x)S(x) = A(x) & \leftrightarrow s = \sum a \\
C(x) = A(x)B(x) & \leftrightarrow c_k = \sum_{j=0}^{k} a_j b_{k-j}, c = a \oplus b
\end{align*}
\]