8.4 Funktioner

Ex 13
3) Grävsverden
Grafen för $z = f(x,y) = \frac{y}{x+y}$ är en hyperbolskiva.

Ex 14
1) Rita grafen för $z = f(x,y) = x^2 + y^2$

Lösning:

Sedan $\mathbb{R}^3$ = $\mathbb{R}^2$ x $\mathbb{R}$

$\mathbb{R}^2$ = $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$

En plan med normalvänster $y'$ och planerad $y = 0$.

För $z = 1$ får $x^2 + y^2 = 1$ (cirkel) En plan med normalvänster $x'$ och planerad $x = 0$.

Lösning 2:

Rita grafen för $z = f(x,y) = \frac{y}{x+y}$.
On the other hand, let us consider again

\[ C \left( \chi, h, c \right) = \frac{h}{h - c} + \frac{c}{h} = 1 \]

be the ellipse

\[ \Gamma \left( x, y \right) = c \quad \Rightarrow \quad x = y - c \]

Einstein's \( \Gamma \left( x, y \right) = 2 \) and

\[ \Gamma \left( x, y \right) = 0 \quad \Rightarrow \quad \text{No inner curve, } \Gamma \left( x, y \right) = 0 \]

Set \( z = \mathcal{F} \in [0, 1] \),

\[ h \in \mathbb{R} \quad \Rightarrow \quad \exists \, \text{Solution} \]

\[ e^x = \text{Shower} \quad z = \mathcal{F} \left( x, y \right) = h - \frac{h}{c} \]
\[ y^2 + x^2 = (y - c)^2 \]

Let \( z = x^2 + y^2 \). Recall that each slice is a

**Ex 16:** If \( R \) is a region in \( \mathbb{R}^2 \) and \( f : \mathbb{R}^2 \to \mathbb{R} \) is a function, then \( \nabla f \) is a vector-valued function that assigns to each point \( (x, y) \) its gradient vector.

On \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \), a function \( f : \mathbb{R}^n \to \mathbb{R} \) is called **smooth** if its partial derivatives exist and are continuous.
Let's solve the given trigonometric problem:

\[
\begin{align*}
\theta &= \Theta \\
\cos \theta &= \frac{x}{r} \\
\sin \theta &= \frac{y}{r} \\
\end{align*}
\]

For any point on the unit circle, the coordinates are given by:

\[
\begin{align*}
x &= \cos \theta \\
y &= \sin \theta \\
\end{align*}
\]

Ex. 1: Find the limit of \( f(x, y) = \frac{x^2 + y^2}{x^2 - y^2} \) as \( (x, y) \to (a, b) \).

Ex. 2: Find the limit of \( f(x, y) = \frac{x^2 y}{x^2 + y^2} \) as \( (x, y) \to (0, 0) \).

To verify a function is differentiable, we have the following:

- \( f(x, y) \) is continuous at \( (a, b) \).
- \( \partial f(x, y) \) exists at \( (a, b) \).
- \( \partial f(x, y) \) is continuous at \( (a, b) \).

For this function, let's find the partial derivatives:

- \( \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left( \frac{x^2 y}{x^2 + y^2} \right) \\
- \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left( \frac{x^2 y}{x^2 + y^2} \right) \\

To find the tangent plane at a point, we use the formula:

\[
z = f(a, b) + \nabla f(a, b) \cdot (x - a, y - b)
\]

For the given function, we have:

- \( f(a, b) = \frac{a^2 b}{a^2 + b^2} \)
- \( \nabla f(a, b) = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) \)

Thus, the equation of the tangent plane at \( (a, b) \) is:

\[
z = \frac{a^2 b}{a^2 + b^2} + \left( \frac{\partial f}{\partial x} \right) (x - a) + \left( \frac{\partial f}{\partial y} \right) (y - b)
\]

Rotation of the plane by \( \theta \):

- \( z = h - y \sqrt{h-y} \)
- \( z = h - x \sqrt{h-x} \)

Alt. case: \( z = \sqrt{h-x} \)
8.1.6 K"ontinuitet

\[ \lim_{x \to a^-} f(x) = f(a) \]

\[ \lim_{x \to a^+} f(x) = f(a) \]

\[ \in D \text{ och } \lim_{x \to a} f(x) = f(a) \]

\[ \text{Der f är K"ontinuerlig i } a \text{ på } D \]