Reforming Mathematics and Science Education

Knut Mørken
Department of Mathematics
Faculty of Mathematics and Natural Sciences
University of Oslo

Öresundsdagen om grundkurser i matematik
på universitet och högskolor
October, 23, 2013
Basic idea

The basic tools for mathematical calculations have changed radically.

How does this influence science education?
Many contributors

Morten Hjorth-Jensen, Dept. of Physics

Hans Petter Langtangen, Dept. of Informatics

Anders Malte-Sørensen, Dept. of Physics

Øyvind Ryan, CMA and Mathematics

Annik Myhre, Solveig Kristensen, Deans of Education

Hanne Sølna, Coordinator of science studies

Scientific staff in mathematics, statistics, physics, astrophysics, ...

Many students
Contents

CSE — Computing in Science Education

Why CSE?
CSE in Oslo
Examples from mechanics
Experiences and challenges
Often said …

Modern industry and technology is impossible without mathematics and science.

Weather forecasting, product design, film production, materials science, cellular phones, iPad, lunar missions, GPS, furniture production, …

Too advanced to discuss here …
Reality

Students hear about the relevance of the sciences

But this relevance is hardly visible in school or the first few years at university

Much emphasis on renewal of the wrapping of science, little on the content
Education

Mathematics → Science/engineering → Realistic applications
Challenge

Education
- Mathematics with pencil and paper

Working life – research
- Calculation by computer
Realistic applications impossible without computing. Requires scientific change! 

Education

Mathematics with computing → Science/eng. with computing → Realistic applications

Impossible without computing
Requires scientific change!
New possibilities with a computer

1 000 000 000 000 000 operations pr. second

- Makes it possible to handle large data sets
- Solution by iteration

The students should learn to exploit and extend this, not just be passive users

Provides an opportunity to expose the students to realistic problems and research early in their studies
Computing

Allows solution of a wide range of problems that cannot be tackled by other methods

Students should learn to exploit and extend this, not just be passive users

Also understanding of limitations

Mastery of computing requires control of the computer – *programming* in combination with mathematics
Implementation of CSE in Oslo

Computational perspective in elementary education within the mathematical sciences:

• Common foundation for 4+ bachelor programmes (250–300 students)
• Computational perspective included in later courses
• Coordinated use of computing in statistics, physics, meteorology, astrophysics, …
# Bachelor programme in physics

<table>
<thead>
<tr>
<th>6. semester</th>
<th>Specialisation</th>
<th>Free</th>
<th>Free</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. semester</td>
<td>Specialisation</td>
<td>FYS2160</td>
<td>ExPhil</td>
</tr>
<tr>
<td>4. semester</td>
<td>FYS2140</td>
<td>FYS2130</td>
<td>FYS1210/FYS2150</td>
</tr>
<tr>
<td>3. semester</td>
<td>MAT1120</td>
<td>FYS1120</td>
<td>AST1100/GEF1000</td>
</tr>
<tr>
<td>2. semester</td>
<td>MAT1110</td>
<td>MEK1100</td>
<td>FYS-MEK1110</td>
</tr>
<tr>
<td>1. semester</td>
<td>MAT1100</td>
<td>MAT-INF1100</td>
<td>INF1100</td>
</tr>
<tr>
<td></td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
</tr>
</tbody>
</table>
## Bachelor programme in physics

<table>
<thead>
<tr>
<th>Semester</th>
<th>Specialisation</th>
<th>5. semester</th>
<th>4. semester</th>
<th>3. semester</th>
<th>2. semester</th>
<th>1. semester</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>Specialisation</td>
<td>Free</td>
<td>Free</td>
<td>FYS2140</td>
<td>MAT1120</td>
<td>MAT1100</td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td>FYS2160</td>
<td>FYS2130</td>
<td>FYS1120</td>
<td>MEK1100</td>
<td>MAT-INF1100</td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td>FYS2150</td>
<td>FYS1210/FYS2150</td>
<td>AST1100/GEF1000</td>
<td>FYS-MEK1110</td>
<td>INF1100</td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td>Free</td>
<td>Free</td>
<td>MAT1110</td>
<td>MEK1100</td>
<td>MAT-INF1100</td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td>Free</td>
<td>Free</td>
<td>MAT1100</td>
<td>MEK1100</td>
<td>INF1100</td>
</tr>
<tr>
<td>1.</td>
<td></td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
</tr>
</tbody>
</table>

10 stp
Foundation

MAT1100 — Calculus:

• Complex numbers, convergence, continuity, differentiation, integration, vector algebra, functions of several variables
# Bachelor programme in physics

<table>
<thead>
<tr>
<th>Semester</th>
<th>Specialisation</th>
<th>4. semester</th>
<th>3. semester</th>
<th>2. semester</th>
<th>1. semester</th>
<th>6 stp</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. semester</td>
<td>Specialisation</td>
<td>FYS2160</td>
<td>ExPhil</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. semester</td>
<td>FYS2140</td>
<td>FYS2130</td>
<td>FYS1210/FYS2150</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. semester</td>
<td>MAT1120</td>
<td>FYS1120</td>
<td>AST1100/GEF1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. semester</td>
<td>MAT1110</td>
<td>MEK1100</td>
<td>FYS-MEK1110</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. semester</td>
<td>MAT1100</td>
<td>MAT-INF1100</td>
<td>INF1100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. semester</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
INF1100 — Introduction to programming with

- Programming in Python, examples from mathematics and science, Matlab-style programming, object orientation, subclasses, inheritance etc.

- Numerical methods, simple graphics, simple Monte Carlo simulation, vector arithmetic, …
## Bachelor programme in physics

<table>
<thead>
<tr>
<th>Semester</th>
<th>Specialisation</th>
<th>3rd Semester</th>
<th>4th Semester</th>
<th>5th Semester</th>
<th>6th Semester</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. semester</td>
<td>MAT1100</td>
<td>MAT-INF1100</td>
<td>INF1100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. semester</td>
<td>MAT1110</td>
<td>MEK1100</td>
<td>FYS-MEK1110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. semester</td>
<td>MAT1120</td>
<td>FYS1120</td>
<td>AST1100/GEF1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. semester</td>
<td>FYS2140</td>
<td>FYS2130</td>
<td>FYS1210/FYS2150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. semester</td>
<td>Specialisation</td>
<td>FYS2160</td>
<td>ExPhil</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. semester</td>
<td>Specialisation</td>
<td>Free</td>
<td>Free</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **10 stp**
Foundation

MAT-INF1100 — Modelling and computations

• Induction, difference equations, Taylor polynomials, differential equations

• Numerical differentiation, integration, interpolation, solution of (systems of) differential equations

• Representation of numbers, characters, sound, images; round-off errors, lossless compression
### Bachelor programme in physics

<table>
<thead>
<tr>
<th>Semester</th>
<th>Specialisation</th>
<th>5. semester</th>
<th>4. semester</th>
<th>3. semester</th>
<th>2. semester</th>
<th>1. semester</th>
<th>10 stp</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>Specialisation</td>
<td>Free</td>
<td>Free</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>FYS2160</td>
<td>ExPhil</td>
<td>FYS2140</td>
<td>FYS2130</td>
<td>FYS1120</td>
<td>FYS-MEK1110</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>FYS1210/FYS2150</td>
<td></td>
<td>FYS1120</td>
<td>AST1100/GEF1000</td>
<td>MEK1100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>MAT1120</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>MAT1110</td>
<td></td>
<td>MEK1100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>MAT1100</td>
<td>INF1100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
<td></td>
</tr>
</tbody>
</table>
Foundation

MAT1110 — Calculus and linear algebra

- Functions of several variables, multiple integrals, elementary linear algebra, linear systems of equations, max/min problems, contractions and completeness, Newton’s method in several variables, power series
- Computations with Matlab or Python
# Bachelor programme in physics

<table>
<thead>
<tr>
<th>Semester</th>
<th>Valgfri fordypning</th>
<th>Valgfritt</th>
<th>Valgfritt</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Valgfri fordypning</td>
<td>FYS2160</td>
<td>ExPhil</td>
</tr>
<tr>
<td>4.</td>
<td>FYS2140</td>
<td>FYS2130</td>
<td>FYS1210/FYS2150</td>
</tr>
<tr>
<td>3.</td>
<td>MAT1120</td>
<td>FYS1120</td>
<td>AST1100/GEF1000</td>
</tr>
<tr>
<td>2.</td>
<td>MAT1110</td>
<td>MEK1100</td>
<td>FYS-MEK1110</td>
</tr>
<tr>
<td>1.</td>
<td>MAT1100</td>
<td>MAT-INF1100</td>
<td>INF1100</td>
</tr>
<tr>
<td></td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
</tr>
</tbody>
</table>
Foundation

MAT1120 — Linear algebra

• Abstract vector spaces, eigenvalues/vectors, orthogonality and least squares, symmetric matrices and quadratic forms
• Markov chains, the power method, simple Fourier analysis, singular value decomposition
• Google page rank, compression of images, wavelets, …
# Bachelor programme in physics

<table>
<thead>
<tr>
<th>Semester</th>
<th>Specialisation</th>
<th>4. semester</th>
<th>5. semester</th>
<th>6. semester</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. semester</td>
<td>MAT1100</td>
<td>MAT-INF1100</td>
<td>-INF1100</td>
<td>Free</td>
</tr>
<tr>
<td>2. semester</td>
<td>MAT1110</td>
<td>MEK1100</td>
<td>FYS-MEK1110</td>
<td>Free</td>
</tr>
<tr>
<td>3. semester</td>
<td>MAT1120</td>
<td>FYS1120</td>
<td>AST1100/GEF1000</td>
<td>Free</td>
</tr>
<tr>
<td>4. semester</td>
<td>FYS2140</td>
<td>FYS2130</td>
<td>FYS1210/FYS2150</td>
<td>Free</td>
</tr>
<tr>
<td>5. semester</td>
<td>Specialisation</td>
<td>FYS2160</td>
<td>ExPhil</td>
<td>Free</td>
</tr>
<tr>
<td>6. semester</td>
<td>Specialisation</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Semester</th>
<th>Mat</th>
<th>Credits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. semester</td>
<td>10</td>
<td>stp</td>
</tr>
<tr>
<td>2. semester</td>
<td>10</td>
<td>stp</td>
</tr>
<tr>
<td>3. semester</td>
<td>10</td>
<td>stp</td>
</tr>
<tr>
<td>4. semester</td>
<td>10</td>
<td>stp</td>
</tr>
<tr>
<td>5. semester</td>
<td>10</td>
<td>stp</td>
</tr>
<tr>
<td>6. semester</td>
<td>10</td>
<td>stp</td>
</tr>
</tbody>
</table>
Bachelor programme in physics

<table>
<thead>
<tr>
<th>6. semester</th>
<th>Specialisation</th>
<th>Free</th>
<th>Free</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. semester</td>
<td>Specialisation</td>
<td>FYS2160</td>
<td>ExPhil</td>
</tr>
<tr>
<td>4. semester</td>
<td>FYS2140</td>
<td>FYS2130</td>
<td>FYS1210/FYS2150</td>
</tr>
<tr>
<td>3. semester</td>
<td>MAT1120</td>
<td>FYS1120</td>
<td>AST1100/GEF1000</td>
</tr>
<tr>
<td>2. semester</td>
<td>MAT1110</td>
<td>MEK1100</td>
<td>FYS-MEK1110</td>
</tr>
<tr>
<td>1. semester</td>
<td>MAT1100</td>
<td>MAT-INF1100</td>
<td>INF1100</td>
</tr>
<tr>
<td></td>
<td>10 stp</td>
<td>10 stp</td>
<td>10 stp</td>
</tr>
</tbody>
</table>
Example
Differentiation

- **MAT1100**
  \[ f'(a) = \lim_{{h \to 0}} \frac{f(a + h) - f(a)}{h} \]

- **MAT-INF1100**
  \[ \frac{f(a + h) - f(a)}{h} \approx f'(a) + \frac{f''(\xi_h)}{2h} + \frac{f(a + h)\epsilon_2 - f(a)\epsilon_1}{h} \]

- **INF1100**
  \[ df_i = \frac{f(a + 10^{-i}) - f(a)}{10^{-i}}, \quad i = 1, 2, \ldots, n \]
Example
Differentiation

0.237976
0.196136
0.125732
0.474854
0.474854
0.294189
0.469055
0.469055
0.433350
0.433350
0.470581
0.470581
0.875549

288 000 numbers
Example
Differentiation

0.237976  0.196136
0.196136
0.196136
0.125732
0.474854
0.474854
0.294189
0.294189
0.469055
0.469055
0.433350
0.433350
0.470581
0.470581
0.875549
Example Differentiation

0.237976  0.196136 – 0.237976
0.196136
0.125732
0.474854
0.294189
0.469055
0.433350
0.470581
0.875549
Example
Differentiation

0.237976  –  0.0418396
0.196136
0.125732
0.474854
0.294189
0.469055
0.433350
0.470581
0.875549
Example
Differentiation

0.237976  – 0.0418396
0.196136  – 0.0704041
0.125732  – 0.0782471
0.474854  – 0.0180664
0.294189  0.0174866
0.469055  – 0.00357056
0.433350  0.00372314
0.470581  0.0404968
0.875549  0.0445557
Applications in mathematics

- Audio and image processing
- Compression
- Ranking of web pages
- ...

Applications in mathematics
Later courses

The maths courses provide a foundation for the use of computing in subjects like physics, astrophysics, meteorology, statistics etc.

No longer necessary that the differential equations are special in order to compute a solution

Makes it possible to study realistic problems
In the 2nd semester mechanics course the students may study ‘advanced’ problems like:

- Simulation of rocket launch, with correct solution provided by data from NASA
- Realistic movement of a football
- New models for friction
Examples from mechanics
Example: Motion of Usain Bolt

- Motion diagram of Usain Bolt
- Natural introduction to numerical derivation:
  \[ \ddot{v}(t) = \frac{x(t + \Delta t) - x(t)}{\Delta t} \]
- Numerical integration of accelerometer readings:
  \[ v(t + \Delta t) = v(t) + a(t) \Delta t \]
  \[ x(t + \Delta t) = x(t) + v(t) \Delta t \]
- Student project: Model race using realistic force laws
  \[ F = F_D + f_c \exp\left(-\left(\frac{t}{t_c}\right)^2\right) - f_v v \]
  \[ F_D = \frac{1}{2} \left(1 - c \exp\left(-\left(\frac{t}{t_c}\right)^2\right)\right) \rho C_D A (v - w)^2 \]
Integration of numerical methods gives simplified exposition of principles

- Introduce robust, applicable problem-solving approaches
- Avoid “marginally solveable” problems
- Demonstrates “Power of Physics”
- Focus on realistic force laws – motivate students to understand their origin
- Constrained motion addressed later

Requires:
- Reorganization
- Simplification
- More advanced force models
Dynamics of Transition from Static to Kinetic Friction

O. M. Braun, I. Barel, and M. Urbakh

1Institute of Physics, National Academy of Sciences of Ukraine, 03028 Kiev, Ukraine
2School of Chemistry, Tel Aviv University, 69978 Tel Aviv, Israel
(Received 29 June 2009; published 6 November 2009)

We propose a model for a description of dynamics of cracklike processes that occur at the interface between two blocks prior to the onset of frictional motion. We find that the onset of sliding is preceded by...
Science in a new framework
Simple applications

May solve a few equations

Paper-pencil
Simple applications

May solve a few equations

Computing
Simple applications

May solve many equations

Computing
Realistic applications

May solve many equations

Computing
Teaching materials

- Hans Petter Langtangen: A Primer on Scientific Programming with Python, 2009
- First semester programming course: http://www.uio.no/studier/emner/matnat/ifi/INF1100/h13/
- Mathematical modelling course http://www.uio.no/studier/emner/matnat/math/MAT-INF1100/h13/
- Second semester mathematics course: http://www.uio.no/studier/emner/matnat/math/MAT1110/v13
- Computational Physics I, fifth semester: http://www.uio.no/studier/emner/matnat/fys/FYS3150/h13/
Experiences challenges
The science is easy, broad collaboration can be demanding!
Greatest challenge is not scientific

Human/organisational challenge:

• Agreement about *overarching* goals
• Broad *collaboration* to reach unified (holistic) implementation across traditional barriers
• Little, but efficient bureaucracy

Requires a culture of trust and collaboration
We have to use Mathematica

The students should primarily learn the basic principles of programming and algorithmic problem solving

The tool should be pedagogical and simple so the these principles are clearly visible

There should be broad consensus about the choice of programming language

Therefore we don’t use Mathematica!
Bottom up
Top down

Broad renewal of teaching requires enthusiasts who collaborate and are willing to put in extra time on teaching.

Success requires support from University (Department, Faculty, School) leadership.
What about the students?

Reform of content — must be done whether the students like it or not

Even the best students are challenged

More focus on generic methods — difficult, particularly in the first semester

Students are motivated by realistic examples!

Students learn what they do every day!
Increased syllabus?

Natural to think that introducing a computational perspective will increase the syllabus

- Traditional text book supplemented with computing

Much theory can be developed using computations if included with the theory

- Requires new text books with computations included in a unified way (6+ new books in Oslo)

Mathematics and theory as important as before
Long perspective

The traditional educational programmes have been developed over hundreds of years.

Time is needed to find good computing-based examples and good pedagogical presentations.

The students need an education that lets them work as specialists for 45 years!
Dissemination

Starting to introduce computing in less mathematical subjects like geo sciences and chemistry

The Ministry of Education is pushing for a computing perspective to be adopted at other universities and university colleges
UiO: Faculty of Mathematics and Natural Sciences

Computing in Science Education

A guide for universities and colleges in Norway
Beyond CSE

• Our work with CSE has led to a more holistic approach to education in general
Studentene lykkes ikke

Å endre en akademisk institusjon er blitt sammenlignet med å flytte en kirkegård. Man får ikke mye hjelp av dem som ligger der.

Knut Olav Åmås, kultur-, debatt- og forskningsredaktør

Disse ordene av Bi-rektor Tom Colbjørnsen slo meg i går da jeg leste den lykke tilstandssrapporten for høyere utdanning og så høre utdanningsledere og -politikere diskutere den på Blindern i Oslo.

«Studentene skal lykkes» har vært to regjeringers mål for den store studiereformen i høyere utdanning de siste ti årene. Slik har det slett ikke gått i denne sektoren med 250 000 studenter og doktorstipendiatere og flere tiotts millioner kroner i offentlig finansiering.

Ikke blitt bedre
Et helt tår har studentene levdt med den såkalte Kvalitetsreformen. «Kvalitet går aldri av moten» er den kliseaktige overskriften på kunnskapsminister Kristin Halvorsens (SV) inledning i tilstandssrapporten.

Men kvaliteten på høyere utdanning er ikke blitt bedre, slik også kvalitetssikringsorganet NOKUTs rapport i vinter så tydelig konkluderte.

Kvalitetsreformen inneførte en kort treårig bachelorgrad og en toårig mastergrad, lovet betere oppfølgning av den enkelte student, mer obligatorisk undervisning og flere skriftlige oppgaver.

Ingen kulturendringer
• Switch from
• to

Input factors → Learning environment → Scientific and professional success

Scientific and professional success

Learning environment

Input factors
InterAct
Culture for learning

• Major initiative on education at MN-faculty:
  • Integration and alignment
  • Relational learning environment
  • Staff development and culture
Changes in the scientific syllabus on the present scale is rare — a unique opportunity!

Requires integration, alignment and a culture that can bring out human synergies.